1. Sets

- A **set** is a well-defined collection of objects.
- Sets are usually represented by capital letters A, B, C, D, X, Y, Z, etc. The objects inside a set are called **elements** or **members** of a set. They are denoted by small letters a, b, c, d, x, y, z, etc.
- If a is an element of a set A, then we say that "a belongs to A" and mathematically we write it as "$a \in A$"; if b is not an element of A, then we write "$b \notin A$".
- There are three different ways of representing a set:
    - **Description method:** Description about the set is made and it is enclosed in curly brackets { }.

    For example, the set of composite numbers less than 30 is written as follows:

    {Composite numbers less than 30}

    - **Roster method or tabular form:** Elements are separated by commas and enclosed within the curly brackets { }.

    For example, a set of all integers greater than 5 and less than 9 will be represented in roster form as {6, 7, 8}.

    - **Set-builder form or rule method:** All the elements of the set have a single common property that is exclusive to the elements of the set i.e., no other element outside the set has that property.

    For example, a set L of all integers greater than 5 and less than 9 in set-builder form can be represented as follows:

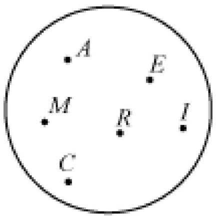
    $L = \{x : x$ is an integer greater than 5 and less than 9$\}$

- **Some important points:**
    1. The order of listing the elements in a set can be changed.
    2. If one or more elements in a set are repeated then the set remains the same.
    3. Each element of the set is listed once and only once.
- On the basis of number of elements, sets can be classified as:
    - **Finite set** − A set that contains limited (countable) number of different elements is called a finite set.
    - **Infinite set** − A set that contains unlimited (uncountable) number of different elements is called an infinite set.
    - **Empty set** − A set that contains no element is called an empty set. It is also called null (or void) set. An empty set is denoted by Φ or {}. Also, since an empty set has no element, it is regarded as a finite set.
- The number of distinct elements in a finite set A is called its **cardinal number**. It is denoted by $n(A)$.
- As the empty set has no elements, therefore, its cardinal number is 0 i.e., $n(\Phi) = 0$

- A set can also be represented using a venn diagram. **Venn diagrams** are closed figures such as square, rectangle, circle, etc. inside which some points are marked. The closed figure represents a set and the points marked inside it represent the elements of the set.

  For example, consider the set of all letters in the word AMERICA. This set consists of the letters A, M, E, R, I, and C.

This set can be represented by a Venn diagram as follows:



- Two finite sets are called equivalent, if they have the same number of elements.

Thus, two finite sets X and Y are equivalent, if $n(X) = n(Y)$. We write it as $X \leftrightarrow Y$ (read as "X is equivalent to Y")

For example, for sets $A = \{-9, -3, 0, 5, 12\}$, $B = \{-2, 1, 2, 4, 7\}$

$n(A) = 5$ and $n(B) = 5$

Therefore, sets A and B are equivalent sets

- Two sets are called equal, if they have same elements.

For example, for sets X = {all letters in the word STONE}, Y = {all letters in the word NOTES}

X = {S, T, O, N, E} and Y = {N, O, T, E, S}

Here, the sets X and Y have same elements. Therefore, in this case, we say that the sets X and Y are equal sets.

- If A and B are any two sets, then set A is said to be a subset of set B if every element of A is also an element of B. We write it as $A \subseteq B$ (read as 'A is a subset of B' or 'A is contained in B'). In this case, we say that B is a **superset** of A. We write it as $B \supset A$ (read as 'B contains A' or 'B is a superset of A').
- If there exists at least one element in A which is not an element of B, then A is not a subset of B.

  Mathematically, we write it as $A \not\subseteq B$.
- Let A be any set and B be a non-empty set. Set A is called a **proper subset** of B if and only if every member of A is also a member of B,and there exists at least one element in B which is not a member of A. We write it as $A \subset B$. Also, B is called the **superset** of A.
- Some important points:

  (a) Every set is a subset of itself.

  (b) A subset which is not a proper subset is called an improper subset. If A and B are two equal sets, then A and B are improper subsets of each other.

  (c) Every set has only one improper subset and that is itself.

  (d) An empty set is a subset of every set.

  (e) An empty set is a proper subset of every set except itself.

  (f) Every set is a subset of the universal set.

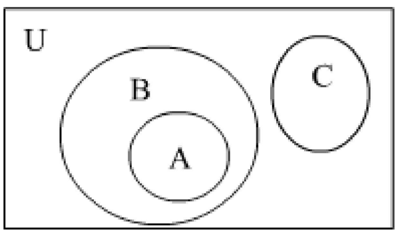
  (g) If $X \subseteq Y$ and $Y \subseteq X$, then $X = Y$

- If cardinal number of the set A is m, i.e., $n(A) = m$, then

  The number of subsets of $A = 2^m$

  The number of proper subsets of $A = 2^m - 1$

- The collection of all subsets of a set A is called the **power set** of A. It is denoted by P(A). In P(A), every element is a set.
- If the number of elements in set A is m, then the number of elements in the power set of A is 2^m.
  i.e., $n\text{P}(A) = 2^m$, where $n(A) = m$
- A set that contains all the elements under consideration in a given problem is called **universal set** and it is denoted by U or ξ.
- Representing information using venn diagram:

  Here, U, A, B and C are four sets.

  From the diagram, following information is observed:

  $A \subseteq B$ or $B \supseteq A$

  Since $B \neq A$, $A \subset B$.

  $C \not\subseteq B$ and $C \not\subseteq A$.

  U is the universal set.

- The union of two sets A and B is the set that consists of all the elements of A, all the elements of B, and the common elements taken only once. The symbol '∪' is used for denoting the union.

  For example, if $X = \{2, 4, 6, 8, 10\}$ and $Y = \{4, 8, 12\}$, then the union of X and Y is given by $X \cup Y = \{2, 4, 6, 8, 10, 12\}$

- There are some properties of union of two sets:
  1. $A \cup B = B \cup A$
  2. $A \cup \Phi = A$
  3. $A \cup A = A$
  4. $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
  5. $U \cup A = U$ (Law of universal set, U)

- The intersection of sets A and B is the set of all elements that are common to both A and B. The symbol '∩' is used for denoting the intersection.

  For example, if $X = \{$A, E, I, O, U$\}$ and $Y = \{$A, B, C, D, E$\}$, then the intersection of the sets X and Y is given by $X \cap Y = \{$A, E$\}$

- The properties of the intersection of two sets are given as follows:

1. $A \cap B = B \cap A$
2. $\Phi \cap A = \Phi$
3. $A \cap A = A$
4. $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)
5. $U \cap A = A$ (Law of U)

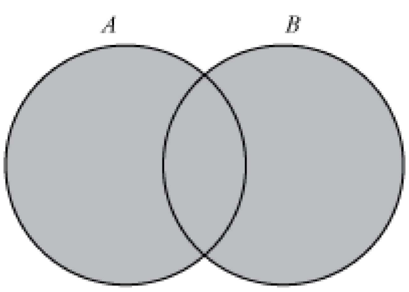
- Two sets are called overlapping (or joint) sets, if they have at least one element in common.
- If two sets A and B are such that $A \cap B = \Phi$ i.e., they have no element in common, then A and B are called disjoint sets.
- **$n (A \cup B) = n (A) + n (B) - n (A \cap B)$**

  If A and B are two disjoint sets i.e., $A \cap B = \Phi$, then $n (A \cap B) = 0$

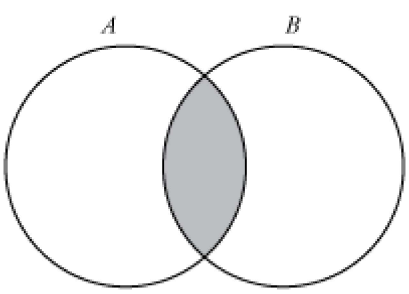
  In this case, the above formula will change into:

  $n (A \cup B) = n (A) + n (B)$

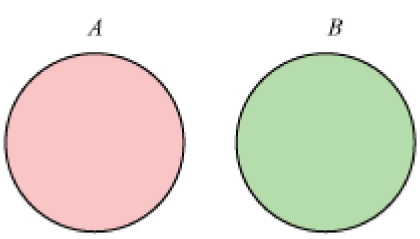
- Venn-diagram for union and intersection of sets are as follows:
  - When the sets A and B are overlapping, the Venn diagram representing $A \cup B$ can be shown as:



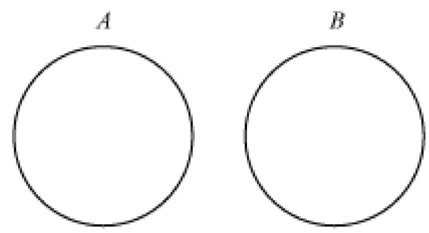
  - When the sets A and B are overlapping, the set $A \cap B$ is the shaded portion of the following the Venn diagram.



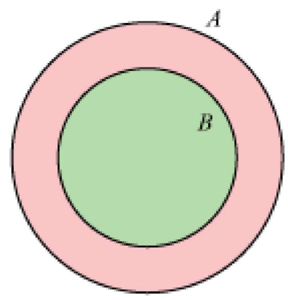
  - When the sets A and B are disjoint, the Venn diagrams representing $A \cup B$ can be shown as:



  - When the sets A and B are disjoint, the Venn diagrams representing $A \cap B$ can be shown as:



- When set B is fully contained in set A, the Venn diagrams representing $A \cup B$ can be shown as:



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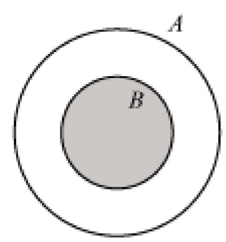
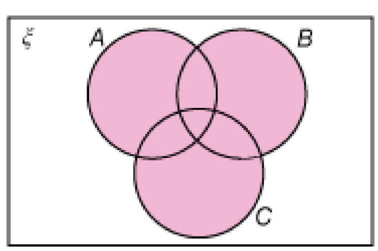
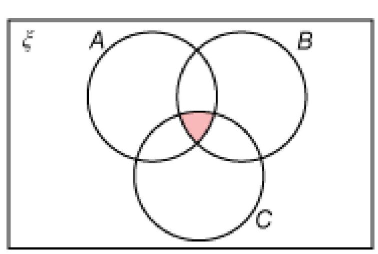


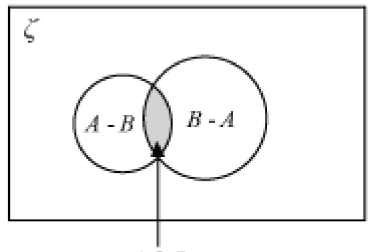
- The union of the three sets A, B and C, i.e., $A \cup B \cup C$, is represented by the shaded portion of the following Venn diagram.



- The intersection of the three sets A, B and C, i.e., $A \cap B \cap C$ is represented by the shaded portion of the following Venn diagram.



- The difference between sets A and B (in that order), i.e., $A - B$ is the set of elements belonging to A, but not to B. Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$.
- If U is the universal set for the sets A, B and C, then the sets $A - B$, $A \cap B$ and $B - A$ can be shown diagrammatically as



- If A and B are two sets, the their symmetric difference is $(A - B) \cup (B - A)$ and denoted by $A \Delta B$. Thus, $A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}$.
- If A and B are two sets, then

  1. $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$
  2. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

- Let X be any set and ξ be its universal set. The complement of set X is the set consisting of all the elements of ξ, which do not belong to X. It is denoted by X' or X^c (read as complement of set X).

  Thus, $X' = \{x | x \in \xi \text{ and } x \notin X\}$ or $X' = \xi - X$

- $n(X') = n(\xi) - n(X)$
- Properties of a set and its complement.

  (a) $X \cap X' = \phi$

  (b) $X \cup X' = \xi$

  (c) $\xi' = \phi$

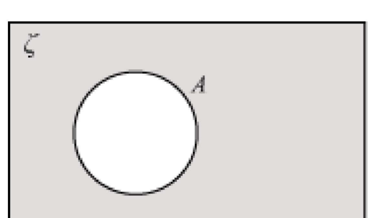
  (d) $\phi' = \xi$

- **De Morgan's laws:**

  (a) $(A \cap B)' = A' \cup B'$

  (b) $(A \cup B)' = A' \cap B'$

- Compliment of a set, A denoted by A^c can be shown in Venn-diagram as follows:



A^C (Shaded portion)

The portion outside the set A, but inside the set ξ, represents the set A^c.